The International Young Researchers' Conference
DOI: doi.org/10.34614/iyrc0051

Acoustic ray propagation in an ellipsoid

Marlyn Li, Raphael Mok
Brookline High School, United States
li_marlyn@yahoo.com

Abstract

Concert hall designs have long been contested in acoustical engineering circles due to the multitude of factors that need to be considered in their geometry. With the increasing digitization of music comes the increased need for immersive recording techniques. Traditional recording techniques involve placing a microphone on or near an instrument, which presents several compromises to sound because instruments are largely designed to be experienced at a distance so that the sound can reverberate through the space it is in. The purpose of this paper is to apply the unique reflective property of light rays within ellipses to sound waves in an ellipsoid. Through using a program to simulate sound waves in an ellipsoid, we show that sound converges at the focus of an ellipsoid. We then make the case for the merits of the use of ellipsoidal geometry in recording applications where the concert hall experience is meant to be simulated.

Keywords: Ellipsoid, foci, pressure waves

Introduction

For centuries, musicians and composers have altered their craft to suit the space they are in. Flowing Gregorian chants were played in cathedrals known for their long reverberation times while classical composers such as Mozart composed delicate articulated pieces to be played in the small well-furnished parlors of the upper echelons. Haydn interestingly used a variety of compositional techniques based on the space he composed for. For example, the fourth movement of Symphony no. 57 is prestissimo, which means extremely fast. This piece premiered in the Esterhàza Music Room, a space known for its rather "dry" sound. Had the venue been larger and therefore had a longer reverberation time, the movement would have sounded rather muddied. In Symphonies no. 102 to 104, Haydn avoided quick dynamic changes. These pieces were composed to premiere at the King's Theatre Concert Hall, a grandiose hall known for its reverberant acoustics. Such a long reverberation time would have rendered quick dynamic changes indiscernible and amplified even quiet sounds rather significantly (Herr, 1998).

Despite this awareness of the effect of space on sound, acoustical knowledge continued to remain limited. It was not until the 1890s that the foundations of acoustic engineering were formed when Wallace Sabine, then a young assistant physics professor at Harvard, began to relate reverberation times to a room's size (Chodos (Ed.), 2011). The formula for reverberation time, which is defined as the seconds it takes for sound to drop 60 decibels, was pioneered by Sabine.

$$Reverberation\ Time = 0.49 \times \frac{Volume}{Surface\ Area \times Absorption\ Coefficient},$$

where volume is given in cubic feet and surface area in square feet.

In concert halls, the goal is to preserve sound emanating from the stage to the audience. Traditionally, this projection is achieved through size or clever hall designs such as wall and ceiling paneling. However, different pieces require different methods of projection. For example, staccato pieces need to maintain a light airy feeling in a “dry” room whereas legato ones need a long reverberation time.

Today, as people increasingly turn to streaming for their listening needs, it becomes more and more necessary to record sound well. From 2015-2020, the number of monthly active users on Spotify, one of the leading music streaming services, increased 470.6% (Statista, 2021). Currently, a common way of recording instruments is “close-miking”, which involves attaching the microphone directly to the body of the instrument. This is widely considered to compromise the true acoustic sound because instruments are largely designed to be experienced at a distance (DPA Microphones, 2015). As a result, for a recording to be the most effective, it is necessary to find a concert hall design that allows for a large concentration of sound upon a single spot. Such a design would allow for an easy installation of a microphone due to sound being centralized whilst still allowing for sound waves to reverberate through the space. It must be noted that an omnidirectional microphone should be used, as it will be able to respond to the total sound pressure and thus best respond to the reflected sound (Rossing & Fletcher, 1995).

Ellipses possess a reflective property where rays originating from one focus will reflect off the ellipse and converge on the other focus (Foster & Pendersen, 2018). Via extrapolation, this property should be true of an ellipsoid. This study aims to show that the reflective property applies to sound waves and show the merits of an ellipsoid as a concert hall design.

Methodology

The application COMSOL Multiphysics was used to simulate the acoustics in an ellipsoid. We applied general wall impedance to the ellipsoid and built-in software air calculations that match those of air in 20°C at 50% humidity. For simplification purposes, the hall had a flat floor section, all walls were smooth, and there were no people or seats.

The ellipsoid used for modeling had a major axis of length 10 and a minor axis of length 6, where (0,0,0) was defined as the center of the ellipsoid. The foci of the ellipse were located at (3,0,0) and (-3,0,0). For the simulations, the focus at (-3,0,0) was designated as the origin of sound. However, due to the symmetrical nature of the shape, the origin focus has no effect on results.

Receivers that sense the sound pressure were placed in several different locations. One was placed at the focus at (3,0,0). This will be referred to as the focus receiver. Next, two more were placed on the major axis at (-1,0,0) and (4,0,0). These will be referred to as the close and far receiver, respectively. The final was placed on the z-axis at (0,0,1). This will be referred to as the high receiver. The frequency of the sound wave was set to 500 Hz, which is between a B4 and C5, and comparable to 440 Hz A, a common tuning standard.

The sound pressure level (SPL) was measured across the x-y plane of the ellipsoid where the foci are located. The SPL is a cross section of sound pressure over time as the sound waves dissipate and shows the cumulative sound along the cross section.

Results

Absorption

As seen in Figure 1, only about 5% of the sound is absorbed by the walls in the simulation when using general wall impedance. This percentage generally changes as the hertz value increases. However, the maximum value it reaches is 10%, a value small enough that there will still be significant sound convergence upon

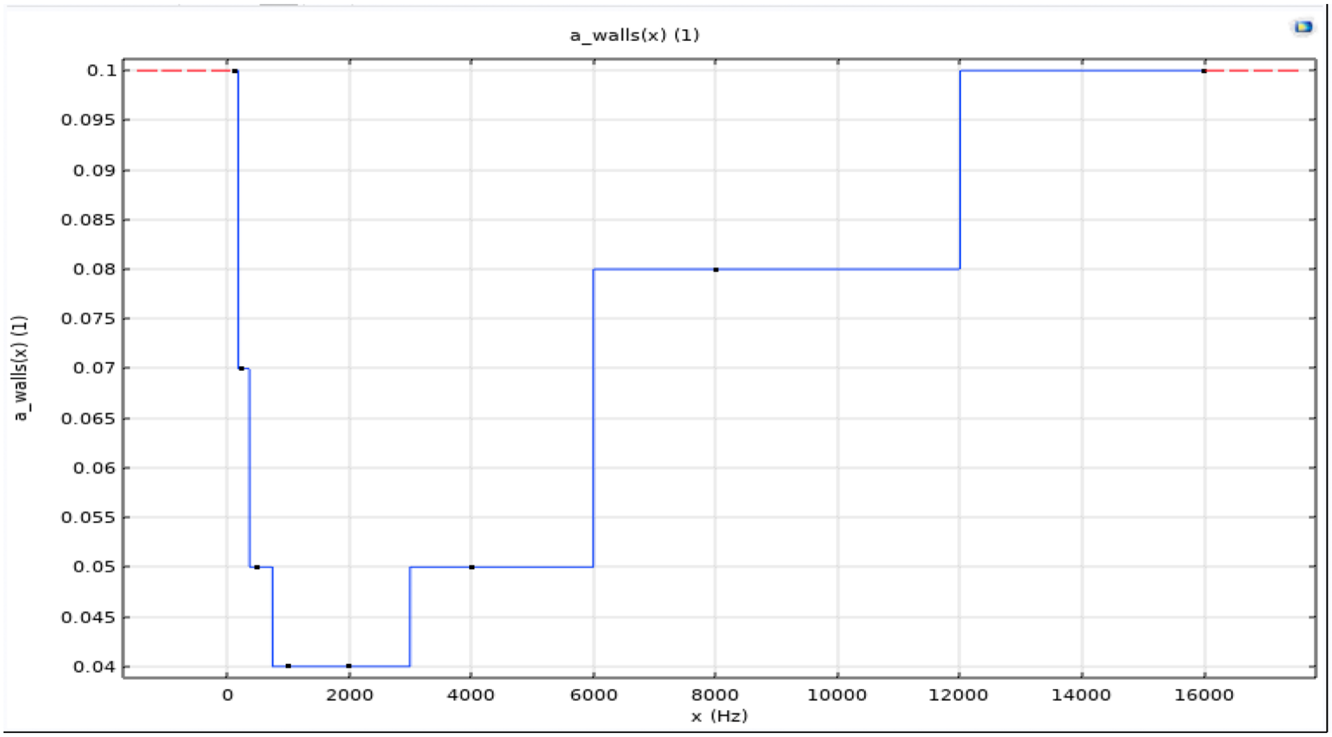

FIGURE 1. The proportion of total sound absorbed by the medium for various Hz of sound (EX: at 2000 Hz, 4% of sound is absorbed).

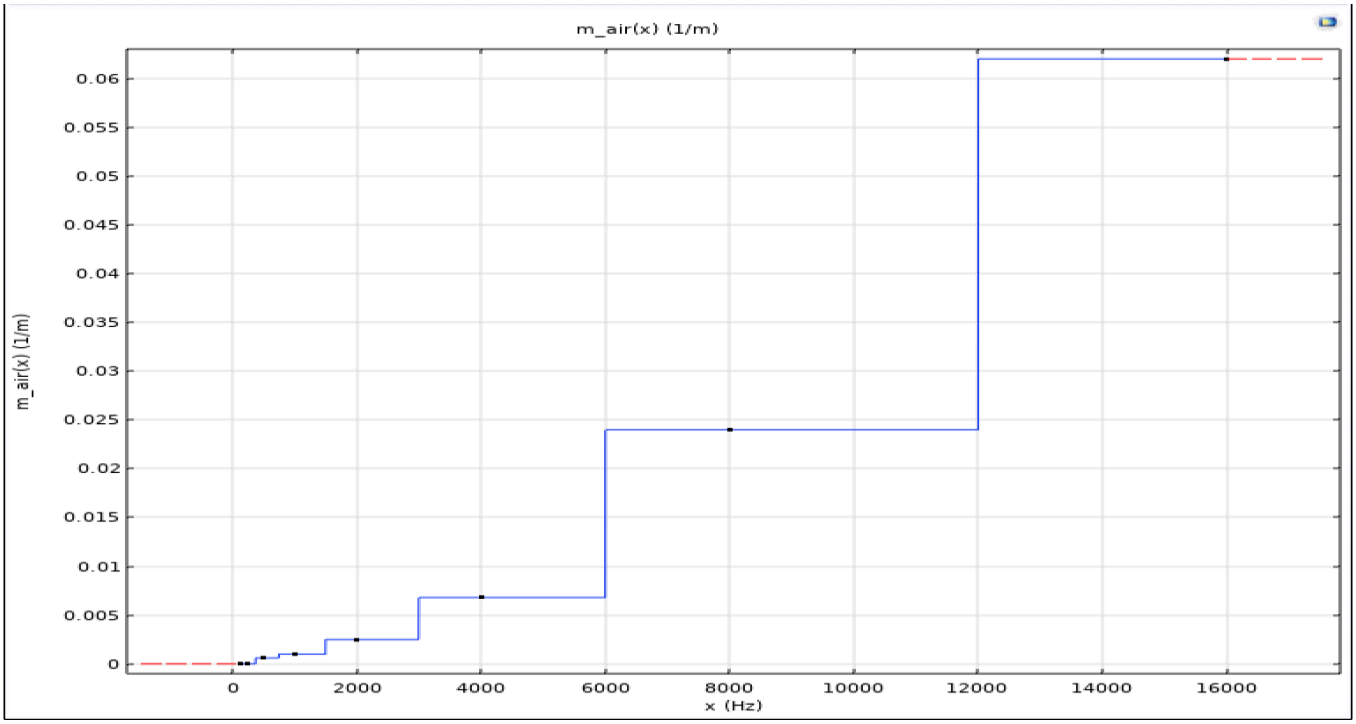

FIGURE 2. Proportion of total sound absorbed by air for different Hz values (EX: at 2000 Hz, 0.25% of sound is absorbed).

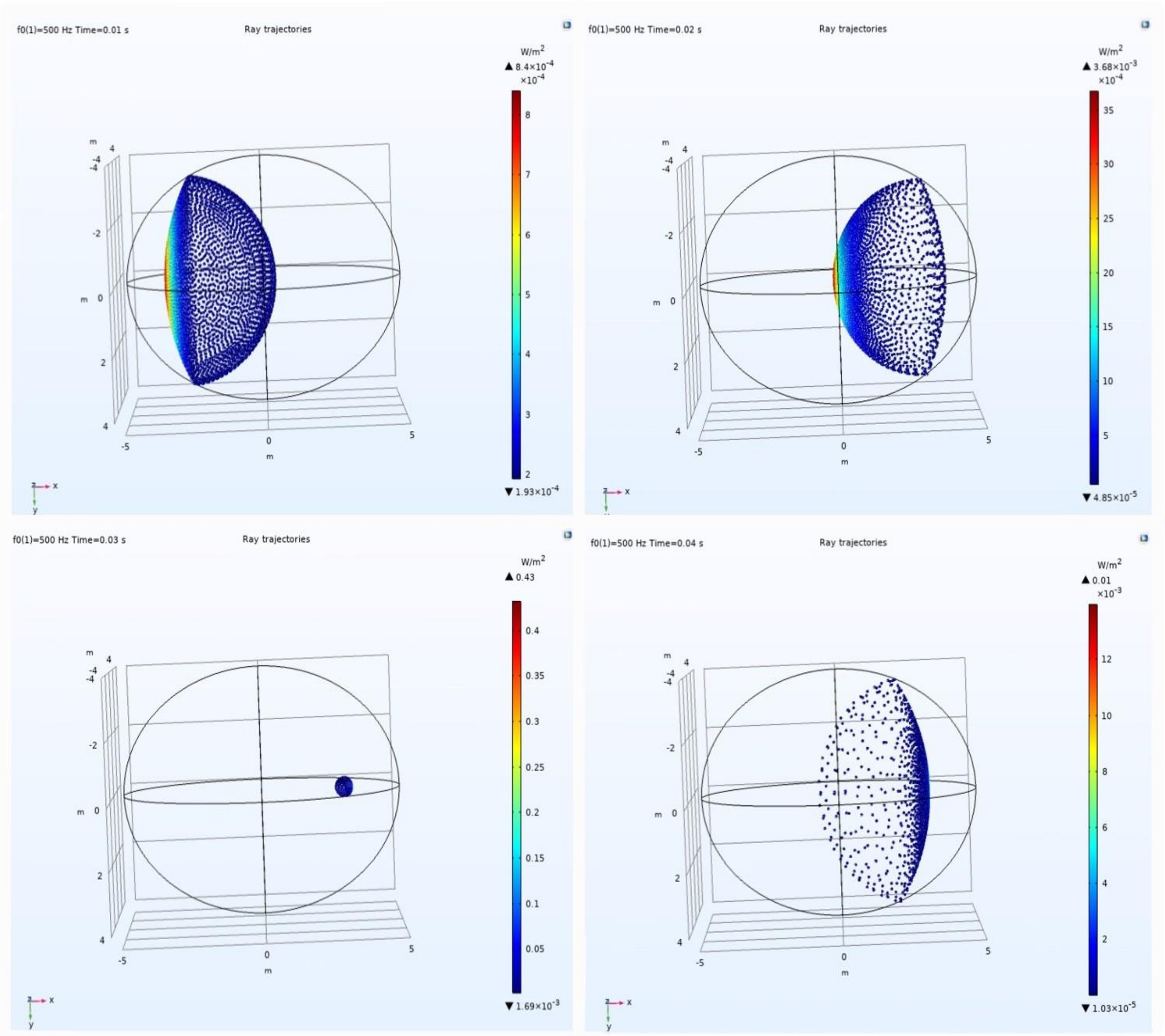

FIGURE 3. Ray trajectories at 0.01-0.04 seconds. Sound is measured in watts per square meter and is represented by colors on the legend to the right. Note: Scales differ between figures pictured.

the focus. As a result, receiving sound at higher frequencies will still be feasible. The absorption through air, as seen in Figure 2, is approximately 0.01% at 500 Hz, meaning it was negligible for the simulation. Even at higher frequencies, air absorption does not surpass 6.3%. Thus, at general air impedance, the strength of the sound converging at the focus will not be affected dramatically either.

Ray Trajectories

As shown in Figure 3, when the ray trajectories were simulated in the ellipsoid in increments of 0.01 seconds, the sound particles converged upon the focus at 0.03 seconds. However, the particles are not completely centralized on the focus and instead are a couple centimeters wide upon the point. This is to be expected because of the different natures of light and sound. Due to sound traveling in longitudinal waves rather than as transverse waves, sound acts as a more general flow of air rather than in a way which can be fully represented as a particle. As a result, the sound "rays" are a representation of the total sound pressure in the air over a minute distance. Thus, the convergence point will not be perfectly centralized. Note that the irradiance of the sound particles upon convergence appears rather low. This is due to the sound losing energy from its initial starting point and the fact that the higher energy particles are located at the center of the orb and are thus not shown in the simulation.

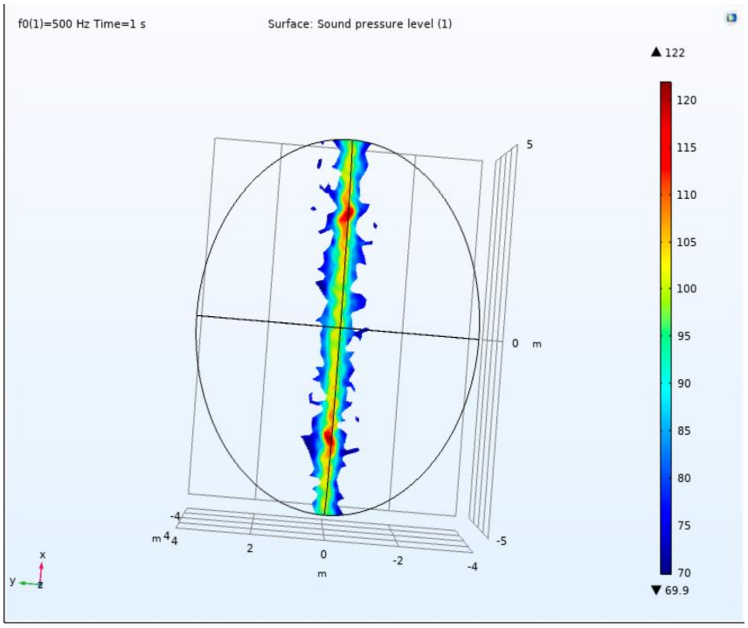

FIGURE 4. Cumulative sound pressure level over x-y plane in decibels at 1 second.

SPL

As shown in Figure 4, the red sections, which show the highest sound pressure, are located at the foci. The sound trails off as it deviates from the major axis so that it eventually becomes negligible. The points on the major axis receive slightly more sound pressure, but it is still significantly lower than that of the foci. The lack of sound in the room apart from along the major axis is not accurately indicative of what would be present in a true hall since a floor and imperfections in the room's surface itself would cause less predictable motion. Such variables would result in more scattering and slight changes in the reflection paths of the sound. However, the acoustic properties of the foci and along the major axis would still be preserved to a major extent. There is also sound present at points on the walls and in the room because the waves must still reflect off the walls. But, due to the spaced-out nature of the sound waves, it is negligible and not recorded by the SPL as it is below the threshold set to idealize the graphics and more adequately show the main results.

Impulse Response

As seen in Figures 5-7, the focus receiver had a superior impulse response to those of the far and high receivers. It is true that a receiver with a closer proximity to the sound source will have a higher impulse response, which can be seen in Figure 8. However, in a practical setting, this receiver would be too close to the stage to place an audience or a microphone. Moreover, although the close receiver received better impulses, it is important to note that it is only capable of receiving sound present on the major axis, meaning there would be significantly less room reverberation on it as compared to the focus. When the close and focus receivers are compared on the SPL, the focus receiver picks up much more net sound because it is receiving sound and room reverberations, both of which are essential to simulate a concert hall setting as closely as possible.

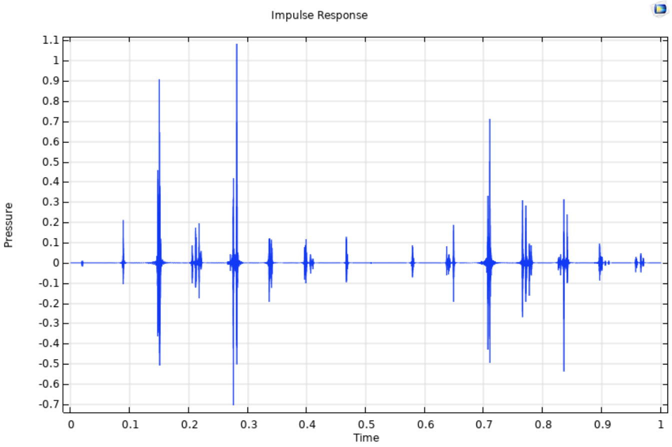

FIGURE 5. Impulse response of far receiver located at (4,0,0). Time is in seconds and pressure is in pascals.

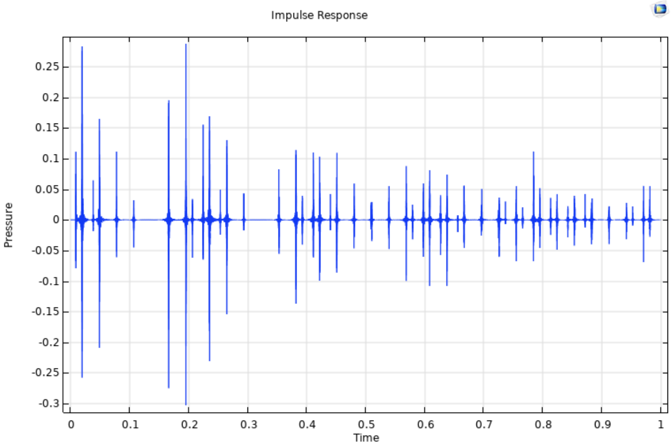

FIGURE 6. Impulse response of high receiver located at (0,0,1). Time is in seconds and pressure is in pascals.

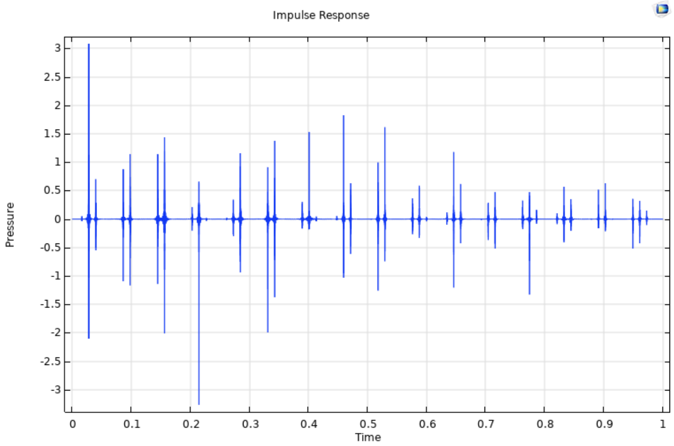

FIGURE 7. Impulse response of focus receiver located at (3,0,0). Time is in seconds and pressure is in pascals.

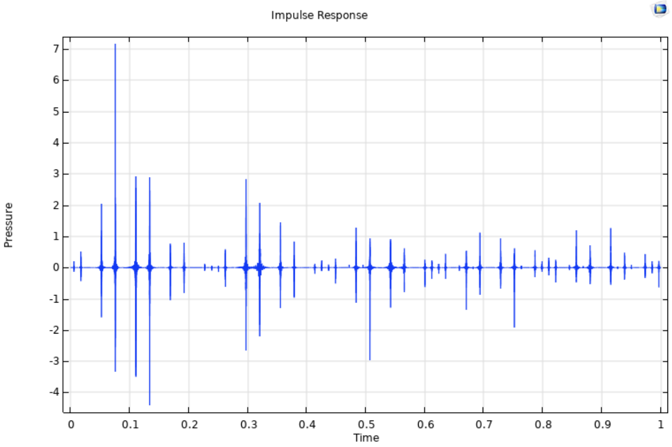

FIGURE 8. Impulse response of close receiver located at (-1,0,0). Time is in seconds and pressure is in pascals.

Discussion

The wall material in the simulation was a general preset and had a general impedance. However, the effect noted in the simulation should occur with any material given that the wall impedance is not high enough to make the sound negligible by the time the sound waves hit the focus. Furthermore, air absorption accounts for such a low percentage of absorbed sound that a change in the room's temperature or humidity would not alter results drastically.

Practical Implications

As people increasingly turn to streaming services for music consumption, the need for a proper recording set up increases. In the past, performers such as Jascha Heifetz recorded pieces with a microphone attached either directly to or close to their instrument (The Newsreel Archive, 2009). This resulted in a very clipped and harsh sound, especially in forte passages requiring heavy contact with the instrument. To produce a more natural sound, the sound produced through the instrument must be allowed to travel the space before being received by a microphone. The fact that the sound waves converge upon one spot in an ellipsoid after reverberating through the space means that such a shape provides an ideal recording environment.

Limitations

This study was conducted in an idealized space devoid of absorption that would normally arise because of seats and people normally present at concerts. Due to the absorption, known as the “seat-dip effect”, there is a loss of intensity of tones between 50 and 500 Hz in traditional concert halls, where the greatest loss occurs at 100 to 150 Hz (Beranek, 1992). However, in a comparison of concert halls in Boston, Vienna, and Amsterdam conducted by Bradley (1991), this seat-dip effect was not significant enough to be noticeable to listeners. Moreover, in a recording environment, many of those sound absorbing obstacles would not be present. The space also lacked a floor to gain the most ideal reflection over the entire ellipsoid. With a floor, the motion would be significantly more complex overall. But, if the ellipsoidal shape is preserved in the roof design, the convergence of the sound reflection on the focus would still be observed to a major degree.

Although the only tested frequency was 500 Hz, the noticed effect will still be true of different frequencies seeing as it is based in a geometric property rather than a specifically acoustic one. The strength of the converging sound will be lower due to the absorption proportion increasing with the frequency, but it will still be easily detectable by a microphone.

Conclusion

Recording in an ellipsoid hall allows for the acoustics of an instrument to be fully realized whilst preserving sound intensity, making it an effective set-up. The field of acoustic engineering is still a developing one. The reflective property of an ellipsoid is promising for use in recording applications, but many variables need to be explored to maximize the output sound whilst preserving concert hall acoustics. The sound energy in a hall is highly correlated with RT/V, where RT is reverberation time in seconds and volume is expressed in m^3. The optimal values of RT/V have been experimentally derived to lie between 100 and 150 x 10^{-6} s/m^3 (Beranek, 2007). Issues arise in acoustical research because architects will never design copies of existing halls. As a result, certain acoustical parameters must be set to predict acoustic quality before the hall is created. The lack of ellipsoid domed concert halls makes it difficult to determine completely if the aforementioned equation pertains to such a shape, although it is highly likely that it does.

The construction materials also have an impact on the acoustics of a hall. Highly porous materials such as carpet are highly effective at absorbing high frequency sound whereas wood absorbs lower frequencies (Wegst, 2006). As a result, a combination of materials must be used. The effects of different material combinations and placements need to be further researched to find the best hall environment for a piece. Moreover, imperfections in placement and human error can result in compromises to the geometry and smoothness of the ellipsoid that effect the reflective property.

The simulation conducted is a proof of concept in showing the acoustical potentials of ellipsoid venues and has implications for solo instruments. However, more exploration is determine needed to the ideal set-up for a larger ensemble, such as an orchestra, where multiple sources of sound are spread out in a much larger area. In this situation, the sound would not reliably begin at a focus. As a result, the sound waves would not converge in as concentrated of an area.

Acknowledgments

We would like to thank the Department of Earth Sciences at the Massachusetts Institute of Technology for providing access to COMSOL Multiphysics.

References

Beranek, L. L. (1992, July 1). *Concert hall acoustics-1992*. https://asa.scitation.org/doi/10.1121/1.404283.

Beranek, L. L. (2007). *Seeking concert hall acoustics*. https://ieeexplore.ieee.org/document/4350232.

Bradley, J. S. (2020, June 6). *A Comparison of Three Classical Concert Halls*. https://nrc-publications.canada.ca/eng/view/object/?id=bf7f26c9-cf79-43b9-b2fc-1749040f50e7.

Chodos, A. (Ed.). (2011, January). *This Month in Physics History*. https://www.aps.org/publications/apsnews/201101/physicshistory.cfm.

DPA Microphones. (2015, October 27). *HOW TO MIC A VIOLIN*. DPA. https://www.dpamicrophones.com/mic-university/how-to-mic-a-violin.

Foster, J. H., & Pederson, J. J. (2018, April 18). *On The Reflective Property of Ellipses*. https://www.tandfonline.com/doi/abs/10.1080/00029890.1980.11995020.

Herr, C. R., & Siebein, G. W. (1998). *An Acoustical History of Theaters and Concert Halls: An Investigation of Parallel Development in Music, Performance Spaces, and the Orchestra*. https://www.acsa-arch.org/proceedings/Annual%20Meeting%20Proceedings/ACSA.AM.86/ACSA.AM.86.29.pdf.

Marshall, A. (2003, July 24). *A note on the importance of room cross-section in concert halls.* https://www.sciencedirect.com/science/article/abs/pii/0022460X67901812.

Rossing, T. D., & Fletcher, N. H. (1995). *Principles of Vibration and Sound*. Springer.

Statista Research Department (2021, January 18). *Spotify MAUs worldwide 2019*. Statista. https://www.statista.com/statistics/367739/spotify-global-mau/.

The Newsreel Archive. (2009). *Caprice #24 di Paganini by Jascha Heifetz*. *YouTube*. https://www.youtube.com/watch?v=pCqv5vm2iz4.

Wegst, U. G. K. (2006, October 1). *Wood for sound*. Botanical Society of America. https://bsapubs.onlinelibrary.wiley.com/doi/pdf/10.3732/ajb.93.10.1439.