On Finding The Focal Length Of Induced Thermal Lens Of Soy Sauce Sample Using Matrix System And Observing The Effect Of Variation Of Parameters

Paras Kumar
M.G.N Public School, Punjab, India
Email: kumarparas218@gmail.com

Abstract- The paper presents the problem in which a Gaussian beam is passed through an absorbing medium which results in heating of the medium which leads to the formation of the thermal lens due to radiative and non-radiative relaxation processes. In this paper, we also discuss the method to find the focal length of the induced thermal lens and by varying parameters such as concentration, focal length, power of the beam. We were able to infer the trend. At last, we were able to spot the effect of gravity in our experimental result during the experiment of the knife-edge method.

INTRODUCTION

Thermal Lens Spectroscopy (TLS) has many industrial and scientific applications; it was first observed by Gordon et. Al[6] and Leite et. Al[5]. TLS is used in measuring concentrations, finding traces of elements in gases and absolute absorption coefficients. Over the decades, scientists have improved the accuracy of finding quantum yields, diffusivity and thermal conductivity of solid media. In our experiment, a laser beam with a Gaussian profile is passed through a thin layer of soy sauce (about $200\mu m$), forming a thermal lens. We chose soy sauce for our experiment as it is a readily-available and affordable liquid capable of producing a thermal lens. It is a fluid that has intense optical absorption in the visual range of the EM spectrum. And it's constituent's solutes have the same linear absorption coefficients. A Gaussian beam from a modulated CW or pulsed laser is focused onto a liquid or solid medium containing fluorophores. After the absorption of the excited beam, the fluorophores at ground energy state are excited to higher energy states, from which they decay by radiative and non-radiative processes. The latter process relies on the transference of heat to the host, thus leading to a time-dependent transverse spatial profile of the temperature $\Delta T(r, t)$ in the sample. The temperature gradient $\Delta T(r, t)$ creates a refractive index gradient($^{dn}/dt$) normal to the beam axis. The resulting gradient $^{dn}/dt$ produces a lens-like object, which is known as a thermal lens (TL), explained by the following equation:

$$n\left(T+\Delta T\right)=n_0+\frac{dn}{dT}\Delta T$$

Briefly, we can say that when light traverse an absorbing medium, the absorbed electromagnetic energy of the beam is partially converted into heat by a non-radiative relaxation process which leads to the heating of medium. This can be explained by the following two aspects: heat equation and diffraction theory. The heat equation predicts the rise in temperature of the medium and the diffraction theory suggests due to rise in temperature of the medium wave in front of a probe beam entering the soy sauce sample undergoes specific modification.

THEORY OF GAUSSIAN BEAM

The Gaussian beam is a beam of monochromatic electromagnetic radiation whose amplitude envelopes in the transverse plane and is given by a Gaussian function. This also implies Gaussian intensity (irradiance) profile, given by:

$$I(r)=I_0e^{-r^2/\omega^2}$$

Where I_0 is the intensity at $r = 0$ and ω_0 is the radial distance at which intensity has decreased to 1/e of its peak value. A lot of information can be concluded about a beam from its wavelength and q parameter. Before introducing the q parameter, we will introduce the beam parameters. The curvature is infinite at z=0. And the beam waist at distance z from the propagation axis can be written as:

$$\omega(z)^2=\omega_0\left[1+\left(\frac{\lambda z}{2\pi\omega_0^2}\right)^2\right]$$

Gaussian beams go through modifications when passed through a cavity or a medium, which will be explained later.

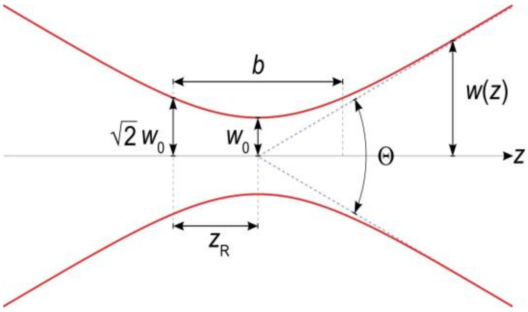


FIGURE 1: The shape of a Gaussian beam


FIGURE 2: Monochromatic Gaussian beam ($\lambda = 532nm$)

The shape of a Gaussian beam of a given wavelength λ is governed solely by the beam waist ω_0(as shown in fig1). The confocal parameter or Rayleigh distance is the distance at which the intensity of the beam is maximum and also at that distance the beam width is 1.414 times the beam waist. We used a gaussian beam with a wavelength of 532nm (as shown in fig2.). We were able to calculate the beam waist of the Gaussian beam by using the knife-edge method, and we were able to plot a Gaussian curve shown in fig3.

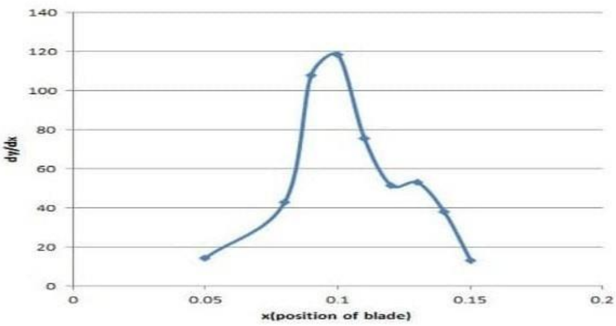


FIGURE 3: Plot of Gaussian beam calculated using knife edge method($\lambda = 532nm$)

EXPERIMENT

When a Gaussian laser beam is passed through a thin sample (about 200$\mu m$to 0.1 mm) of soy sauce; we can observe the thermal lensing effect. We infer a set of concentric rings on the screen.

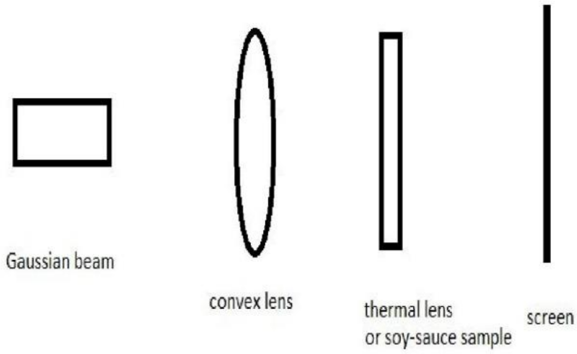


Figure 4: Block diagram of experimental setup

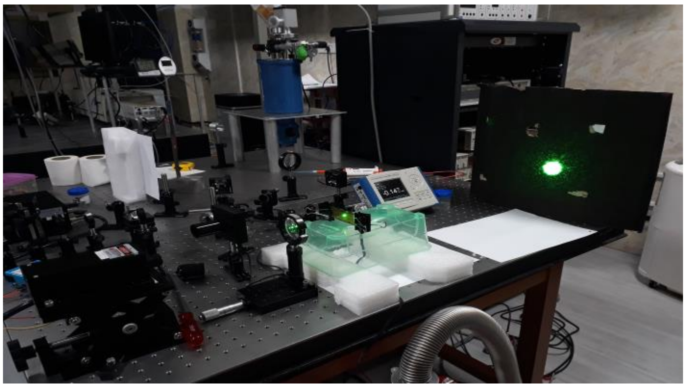


FIGURE 5: Experimental setup.

First of all, we will prepare a sample of soy sauce (100%). Take a few drops of soy sauce on the slide and then gently place the other slide on it and both sides carefully attached using a clip as shown in fig 6. The area with no bubbles was best suited for the experiment. We arranged the equipment (as shown in fig4). Fig5 shows the actual arrangement in our lab. Now the Gaussian beam is passed through the collimator (f = 10cm) to converge the beam and then it is passed through the soy sauce sample .After some time, we could see the thermal lensing effect as concentric rings are formed(as shown in fig7).We can observe that it took some time for these rings to form, which shows in this process Thermal lenses are formed because thermal processes are slower in comparison to nonlinear electric effects. On page1 we have shown why the thermal lens is formed, to surmise it: thermal lens is formed when light passes an absorbing medium such as soy-sauce, it absorbs electromagnetic energy and converts the energy into heat by non-radiative relaxation processes which lead to heating of the medium and further leads to change in the refractive index of the medium. The formation of concentric rings can be explained by diffraction theory and spatial self-phase modulation. We can also explain the formation of the thermal lens by heat exchange theory but the math involved is complicated for e.g. Whinery solution gives us the mathematical expression for the change in heat when a beam is passed through a sample

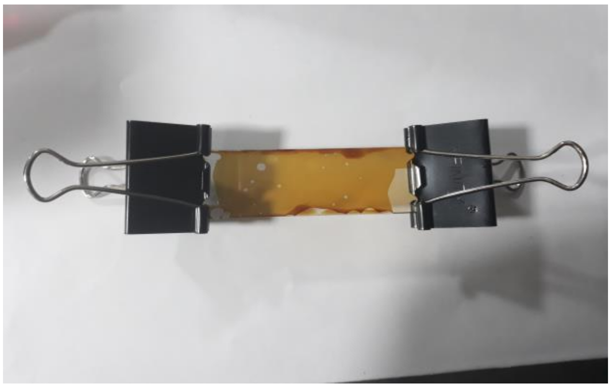


FIGURE 6: Soy sauce sample.

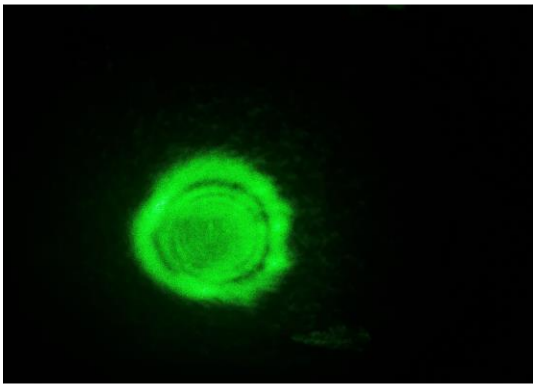


FIGURE 7: Concentric rings formed on screen (N=7).

DIFFRACTION THEORY

According to paper by Gordon [6] in 1965 focal length of induced thermal lens can be written in terms of some physical factors.

$$f = \frac{\pi k \omega_0^2}{2.303 P_e (\frac{dn}{dt}) A}$$

Where A is the area, k is the thermal coefficient, dn/dt is thermo optic coefficient, and P is the power. Now according to him if the Thermo optic coefficient is greater than 0 it will behave as a concave lens, elsewise it behaves as a convex lens. In our experiment, we can see that the soy-sauce sample behaves as a convex lens.

The diffraction theory proposes that change in temperature arises due to the phase delay in the wavefront of the beam. Born and E. Wolf [4] in their book ‘Principles of optics’ describes it as a perturbation due to an additional phase lag to phase beam of a spherical wave. We can see in fig8 the difference between the wavefront before and after hitting the medium. wavefront emerges from the soy-sauce sample having an additional phase lag.

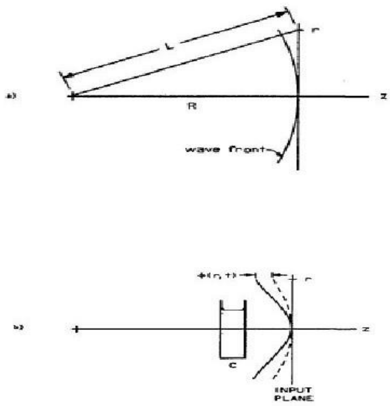


FIGURE 8: a) wave from before entering the soy sauce sample b) wave front with additional phase lag exiting the soy sauce sample for a) and b) the optical path length can be written as:

$$\varphi = nl$$
$$\varphi(r,t) = ln(r,t) - n(0,t)$$

we know,

$$n(r,t) = n_0 - \frac{dn}{dT}\Delta T(r,t)$$

solving this we get,

$$\frac{2\pi}{\lambda}\phi(r,t) = \frac{2\pi}{\lambda}\frac{dn}{dt} l \Delta T(0,t) - \Delta T(r,t)$$

This equation shows us that due to change in phase, there is a change in temperature which leads to change in the refractive index of the medium [2].

CALCULATION OF FOCAL LENGTH

First of all, we need to introduce the q parameter to understand ABCD LAW and how we can calculate the focal length of the induced thermal lens using the matrix method. The q parameter or gaussian complex curvature parameter can be derived from the basic gaussian equations, but we won’t derive the equation here. we can express q in a relation as following

$$\frac{1}{q} = \frac{1}{R} - \iota \frac{\lambda}{\pi \omega_0^2}$$

where R is curvature of beam.

When the beam enters the soy sauce sample the q at the entrance(q_1) and at the exit of sample(q_2) are related by the following equation:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

OR

$$\frac{1}{q_2} = \frac{C + {}^{D}/_{q_1}}{A + {}^{B}/_{q_1}}$$

where A, B, C, D are the elements of the ABCD matrix which are unique for every optical system and can be determined experimentally.

Matrix for a lens can be written as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

Matrix for free space can be written as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

Where l is the distance and f is focal length of the lens.
3.For an optical system the effective matrix can be written by the multiplication of individual matrices.

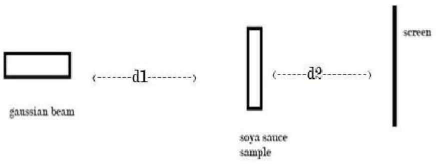


FIGURE 8: Simple block diagram of optical system for e.g., the effective ABCD matrix for optical system described in fig is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \mathrm{d}_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & \mathrm{d}_1 \\ 0 & 1 \end{pmatrix}$$

We will be using this system to calculate the focal length of the soy sauce sample. A convex lens is removed in this experiment to ease the calculation. As we can see that the distance between beam and screen will be affecting the focal length, but we will discuss this in the next section. Solving the equation, we can find an expression for focal length (the proof of the expression is available in the appendix.)

$$f_{th} = \frac{d_1\omega_2^2 - d_2\omega_1^2}{\omega_2^2 - \omega_1^2}$$

In this experimentω_1 is the waist of the beam at the left of the thermal lens, which we can calculate using the knife-edge method. on plugging the values $d_1 = 10.5cm$, $d_2 = 20.45cm\omega_1 = 0.23mm$ and $\omega_2 = 0.31mm$, we get the focal length of the induced thermal lens equal to -1.68cm, which means it behaves like a diverging lens.

VARIATION OF PARAMETERS

I. Number of rings

During the experiment, we noticed that if we vary the power of the beam, we observe a linear change in the no.of rings also, we were able to plot the data on a line graph, a straight line graph was obtained.

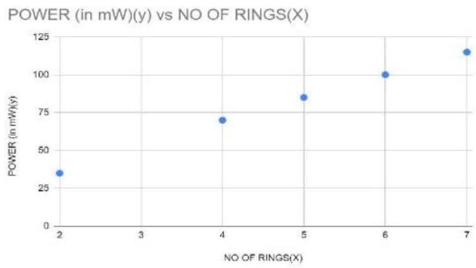


FIGURE 9: We can see no. of rings increases linearly with power (${}^{P}/N \approx 17$)

II. Effect of concentration

According to beer lambert law, intensity of beam after passing through the sample can be written as:

$$\frac{I_0}{I} = 10^{bcl}$$

where b is the coefficient of absorption, c is the concentration and l is the thickness of the sample. we know that

,

$$P = \frac{I\pi\omega^2}{2}$$

using this in equation 3 we get

$$f = \frac{2k}{IA({}^{dn}/_{dt})}$$

which means it is inversely related to I, using beer lambert law we can generalise the equation and rest of the terms A, dn/dt, b, l and k are physical constants and have their usual meanings. so, we can say that $f = m10^{ac}$ where m and a are constant. from here we can expect the relation between concentration and focal length. Similarly, we were able to plot the graph between P and concentration, by modifying the equation (9) and were verified with experimental results as shown below.

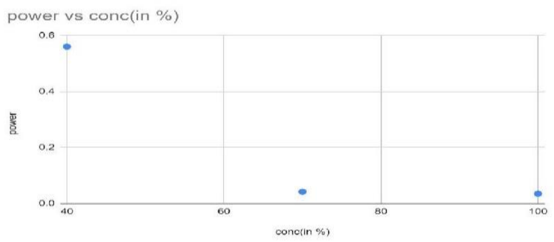
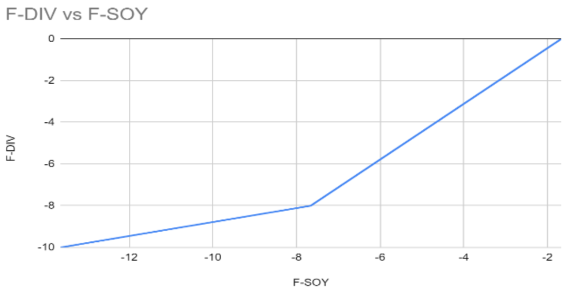


FIGURE 11

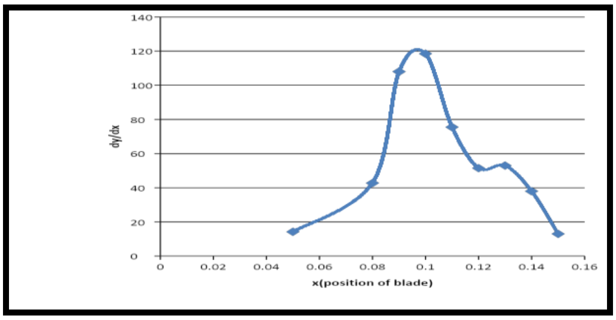
III. Varying the focal length of lens

In this experiment, we used a diverging lens and noticed the change in focal length of soy sauce sample with a change in focal length of the diverging lens. The focal length was calculated using the matrix method. we obtain the following plot.

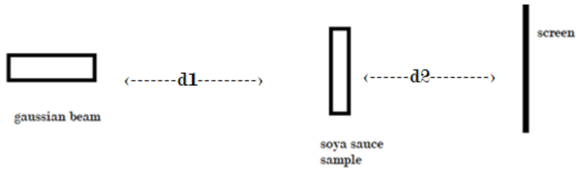


IV. Effect of gravity

This is the most important aspect. As mentioned earlier thermal optical phenomena are slow as compared to other non-linear effects. This means we need to expose the sample for a specific duration of time. Now due to gravity, the sample will naturally flow down changing the concentration of the exposed area, which leads to change in calculations. We were not able to formulate maths behind the effect of gravity, but it is visible from our experiment (as seen in the figure below) that the bump in the Gaussian curve arises due to change in concentration. This curve was drawn during the calculation of ω_2 for calculating the focal length using matrix-method.



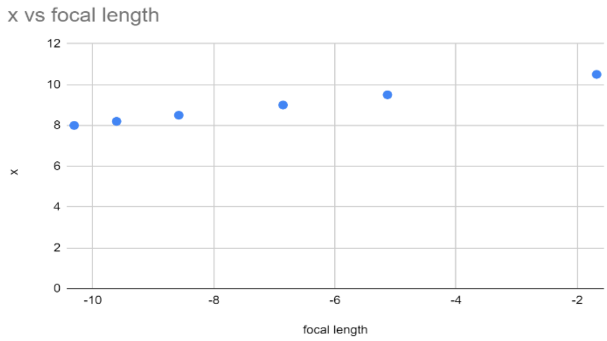
V. The distance x



We know $d_1 + d_2 = d$ and let us assume $d_1 = x$
Putting $d_2 = d - x$ in eq (11)
We get,

$$f = x * \frac{\omega_2^2 + \omega_1^2}{\omega_2^2 - \omega_1^2} - d * \frac{\omega_1^2}{\omega_2^2 - \omega_1^2}$$

We conducted experiment in which we varied the value of x and plotted the following graph



CONCLUSION

In this paper, we proposed the idea of the non-linear effect, which was proposed by Gordon in 1965, using the diffraction theory, we were able to understand the reason behind the change in the refractive index. We also used the matrix method to calculate the focal length of the thermal lens then lastly we varied various parameters to observe their effect on the focal length of the soy-sauce sample for e,g.. gravity, concentration etc.

ACKNOWLEDGMENT

The authors are thankful to Indian institute of science and research Kolkata, Dr BipulPal, National institute of technology Jalandhar and Aryano Ray.

APPENDIX

In eq (8) if,

$$R \to -\infty$$

(because at focus the wave front is flat)

$$\frac{1}{q} = -\iota \frac{\lambda}{\pi \omega_0^2}$$

modifying eq (9) we get,

$$Cq_1q_2 + Dq_2 = Aq_1 + B$$

q_1q_2 will be real, so comparing real and imaginary terms we can say

$$\omega_1^2 = \frac{A}{D}\omega_2^2$$

now modifying eq (10) we get

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - {d_1}/{f} & d_1 + d_2 - {d_1 d_2}/{f} \\ -1/f & 1 - {d_2}/{f} \end{pmatrix}$$

Comparing the two we can say

$$\omega_1^2 = \frac{1 - {d_1}/{f}}{1 - {d_2}/{f}}\omega_2^2$$

and modifying this we get our final answer which we have used to calculate the focal length

$$f_{th} = \frac{d_1\omega_2^2 - d_2\omega_1^2}{\omega_2^2 - \omega_1^2}$$

BIBLIOGRAPHY

1.Turchiello, Rozane de F., Luiz AA Pereira, and Sergio L. G´omez. Low-cost nonlinear optics experiment for undergraduate instructional laboratory and lecture demonstration.” American Journal of Physics 85.7 (2017): 522528

2.Sheldon, S. J., L. V. Knight, and J. M. Thorne.” Laserinduced thermal lens effect: a new theoretical model.” Applied optics21.9 (1982): 1663-1669.

3.Snook, Richard D., and Roger D. Lowe. ” Thermal lens spectrometry. A review.” Analyst 120.8 (1995): 20512068.

4.E. Hecht, Optics, 4th ed. (Addison Wesley, San Francisco, 2002)

5.THE THERMAL LENS EFFECT AS A POWER-LIMITING DEVICE R. C. C. Leite 1967

6.LongTransient Effects in Lasers with Inserted Liquid Samples Gordon 1965

7. C. Hu and J. R. Whinnery, Appl. Opt. 12,72 (1973)